Math 45 6.4 Special Products Day 2

Objectives Remaining: 4) Factor Difference of Two Cubes
5) Factor Sum of Cubes

Chapter 5 Review.

Multiply.

① $(2x-5)(4x^2+10x+25)$

distribute to get six terms.

$$= 8x^3 + 20x^2 + 50x$$
$$\quad - 20x^2 - 50x - 125$$

$$= \boxed{8x^3 - 125}$$

② $(3x+4)(9x^2-12x+16)$

$$= 27x^3 - 36x^2 + 48x$$
$$\quad + 36x^2 - 48x + 64$$

$$= \boxed{27x^3 + 64}$$

Chapter 6 Goal: Start with the end results and factor. Do the chapter 5 problem backward.

To do this we need to get

in ① from $8x^3$ to $2x$ and $4x^2$
from 125 to 5 and 25
plus $10x$ from somewhere.

in ② from $27x^3$ to $3x$ and $9x^2$
from 64 to 4 and 16
plus $-12x$ from somewhere.

$\Rightarrow$ Need cube roots!
(For part of this.)

List Perfect Cubes

1	$1^3 = 1$
2	$2^3 = 8$
3	$3^3 = 27$
4	$4^3 = 64$
5	$5^3 = 125$
6	$6^3 = 216$
7	$7^3 = 343$
8	$8^3 = 512$
9	$9^3 = 729$
10	$10^3 = 1000$

List Perfect Cube Roots.

$\sqrt[3]{1} = 1$

$\sqrt[3]{8} = 2$

$\sqrt[3]{27} = 3$

$\sqrt[3]{64} = 4$

$\sqrt[3]{125} = 5$

$\sqrt[3]{216} = 6$

$\sqrt[3]{343} = 7$

$\sqrt[3]{512} = 8$

$\sqrt[3]{729} = 9$

$\sqrt[3]{1000} = 10$

<u>Note</u>: 64 is on both lists.

$\sqrt[3]{64} = 4$

$\sqrt{64} = 8$

For variables

x	x^3
x^2	$(x^2)^3 = x^6$
x^3	$(x^3)^3 = x^9$
x^4	$(x^4)^3 = x^{12}$
x^5	$(x^5)^3 = x^{15}$

For any x (positive or neg)

$\sqrt[3]{x^3} = x$

$\sqrt[3]{x^6} = x^2$

$\sqrt[3]{x^9} = x^3$

$\sqrt[3]{x^{12}} = x^4$

To take the cube root
Divide exponent by 3.

If exponent is not divisible by 3, the term is not a perfect cube.

To take cube root of a fraction, take cube roots of numerator and denominator

$$\sqrt[3]{\frac{343}{z^6}} = \frac{\sqrt[3]{343}}{\sqrt[3]{z^6}} = \frac{7}{z^2}$$

Math 45 6.4 Day 2 p.3

Factor completely.

(3) $8x^3 - 125$

step 0: Recognize Difference of Two Cubes

- 2 terms
- subtracted
- both are perfect cubes.

step 1: Take the cube root of the first term to find a.

$a = \sqrt[3]{8x^3}$

$a = \sqrt[3]{8} \cdot \sqrt[3]{x^3}$

$a = 2x$

step 2: Take the cube root of the second term to find b. (Ignore negative.)

$b = \sqrt[3]{125}$

$b = 5$

step 3: Substitute a and b into

Difference of Two Cubes Formula

$$\boxed{a^3 - b^3 = (a-b)(a^2 + ab + b^2)}$$

* CAUTION *
Use ( ) around $(a)^2$ and $(b)^2$ so all parts are squared

$= (a - b)(a^2 + ab + b^2)$

$= (2x - 5)((2x)^2 + (2x)(5) + (5)^2)$

step 4: Simplify the trinomial.

$$= \boxed{(2x - 5)(4x^2 + 10x + 25)}$$

(4) $27x^3 + 64$

step 0: Recognize Sum of Two Cubes
- 2 terms
- added
- both are perfect cubes.

step 1: Take cube root of first term to find a.

$a = \sqrt[3]{27x^3}$

$a = \sqrt[3]{27} \cdot \sqrt[3]{x^3}$

$a = 3x$

step 2: Take cube root of second term to find b.

$b = \sqrt[3]{64}$

$b = 4.$

step 3: Substitute a and b into

Sum of Two Cubes Formula
$$\boxed{a^3 + b^3 = (a+b)(a^2 - ab + b^2)}$$

* using parentheses!

$= (a + b)(a^2 - a \cdot b + b^2)$

$= (3x + 4)((3x)^2 - (3x)(4) + (4)^2)$

step 4: Simplify the trinomial.

$= \boxed{(3x+4)(9x^2 - 12x + 16)}$

(5) $384s^3 + 6.$

GCF 6!

$= 6\left(\frac{384s^3}{6} + \frac{6}{6}\right)$

$= 6(64s^3 + 1)$

Sum of cubes

$a = \sqrt[3]{64s^3} = \sqrt[3]{64} \cdot \sqrt[3]{s^3} = 4s$

$b = \sqrt[3]{1} = 1$

$= 6(4s+1)((4s)^2 - (4s)(1) + (1)^2)$

$= \boxed{6(4s+1)(16s^2 - 4s + 1)}$

Mnemonic device for sum and difference of cubes:

Some people use the mnemonic "SOAP" for the signs; the letters stand for "same" as the sign in the middle of the original expression, "opposite" sign, and "always positive".

$$a^3 \pm b^3 = (a \text{ [same sign] } b)(a^2 \text{ [opposite sign] } ab \text{ [always positive] } b^2)$$

Math 45 6.4 Day 2 p. 5

(6) $250n^4 - 54n$

GCF $2n$

$= 2n\left(\frac{250n^4}{2n} - \frac{54n}{2n}\right)$

$= 2n(125n^3 - 27)$

difference of cubes

$a = \sqrt[3]{125n^3} = 5n$

$b = \sqrt[3]{27} = 3.$

$= 2n(5n - 3)((5n)^2 + (5n)(3) + (3)^2)$

$= \boxed{2n(5n-3)(25n^2 + 15n + 9)}$

(7) $8m^3 + 125n^6$

sum of cubes

$a = \sqrt[3]{8m^3} = 2m$

$b = \sqrt[3]{125n^6} = 5n^2$

$= (2m + 5n^2)((2m)^2 - (2m)(5n^2) + (5n^2)^2)$

$= \boxed{(2m + 5n^2)(4m^2 - 10mn^2 + 25n^4)}$

(8) $25a^3 + 81b^6$

coefficients are squares

variables are cubes.

This is neither pattern.

$\boxed{\text{PRIME}}$

(9) $\frac{x^3}{125} + 8$

sum of cubes

$a = \sqrt[3]{\frac{x^3}{125}} = \frac{\sqrt[3]{x^3}}{\sqrt[3]{125}} = \frac{x}{5}$

$b = \sqrt[3]{8} = 2$

$= \left(\frac{x}{5} + 2\right)\left(\left(\frac{x}{5}\right)^2 - \left(\frac{x}{5}\right)(2) + (2)^2\right)$

$= \boxed{\left(\frac{x}{5} + 2\right)\left(\frac{x^2}{25} - \frac{2x}{5} + 4\right)}$

Math 45 6.4 Special Products

1) Difference of Two Squares
   - Two terms
   - Subtracted
   - Both terms are perfect squares

Formula: $a^2 - b^2 = (a-b)(a+b)$ Example: $49x^2y^2 - 81z^2 = (7xy - 9z)(7xy + 9z)$

Step 1: Take square root of first term to get a.
Step 2: Take square root of last term to get b.
Step 3: Substitute a and b into formula.

2) Perfect Square Trinomials
   - Three terms
   - Last term is added
   - First and last terms are perfect squares

Formula: $a^2 - 2ab + b^2 = (a-b)^2 = (a-b)(a-b)$ Example: $\frac{x^2}{25} - \frac{6}{5}x + 9 = \left(\frac{x}{5} - 3\right)^2$

Formula: $a^2 + 2ab + b^2 = (a+b)^2 = (a+b)(a+b)$ Example: $4x^2 + 44x + 121 = (2x+11)^2$

Step 1: Take square root of first term to get a.
Step 2: Take square root of last term to get b.
Step 3: Substitute a and b into formula.
Step 4: Check by FOIL – middle term must be correct. If middle term is not correct, trinomial is prime.

3) Difference of Two Cubes
   - two terms
   - Subtracted
   - First and last terms are perfect cubes

Formula: $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$ Example: $64x^3y^3 - 125z^3 = (4xy - 5z)(16x^2y^2 + 20xyz + 25z^2)$

Step 1: Take cube root of first term to get a.
Step 2: Take cube root of last term to get b. (Ignore negative.)
Step 3: Substitute a and b into formula.

4) Sum of Two Cubes
   - two terms
   - Added
   - First and last terms are perfect cubes

Formula: $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$ Example: $\frac{x^6}{27} + 8y^3 = \left(\frac{x^2}{3} + 2y\right)\left(\frac{x^4}{9} - \frac{2}{3}x^2y + 4y^2\right)$

Step 1: Take cube root of first term to get a.
Step 2: Take cube root of last term to get b. (Ignore negative.)
Step 3: Substitute a and b into formula.

5) Sum of Two Squares
   - two terms
   - Added
   - First and last terms are perfect squares

Sum of two squares is always PRIME. Example: $36x^2 + 4$ is prime.

$\boxed{\text{Math 45}}$ Examples, 6.4 (Day 1)

Factor completely.

① $49x^2y^2 - 81z^2$

- two terms
- subtracted
- both are perfect squares

Difference of two squares

$a^2 - b^2 = (a+b)(a-b)$

$a = \sqrt{49x^2y^2} = 7xy$

$b = \sqrt{81z^2} = 9z$

$a^2 - b^2 = (a-b)(a+b)$

$= \boxed{(7xy - 9z)(7xy + 9z)}$

② $\frac{x^2}{25} - \frac{6}{5}x + 9$

- 3 terms
- first and last are perfect squares
- last is added

Perfect square trinomial

$a^2 - 2ab + b^2 = (a-b)^2$

$a = \sqrt{\frac{x^2}{25}} = \frac{x}{5}$

$b = \sqrt{9} = 3$

$a^2 - 2ab + b^2 = (a-b)^2$

$= \boxed{\left(\frac{x}{5} - 3\right)^2}$

check middle term:

$2(-3)\left(\frac{x}{5}\right) = -\frac{6x}{5}$ ✓

③ $4x^2 + 44x + 121$

- 3 terms
- first and last are perfect squares
- last is added

Perfect Square Trinomial

$a^2 + 2ab + b^2 = (a+b)^2$

$a = \sqrt{4x^2} = 2x$

$b = \sqrt{121} = 11$

$a^2 + 2ab + b^2 = (a+b)^2$

$= (2x + 11)^2$

check middle term:

$2(2x)(11) = 44x$ ✓

Math 45 Examples, 6.4 (Day 2)

Factor completely.

(4) $64x^3y^3 - 125z^3$

- 2 terms
- subtracted
- both are perfect cubes

Difference of Two Cubes.

$a^3 - b^3 = (a-b)(a^2+ab+b^2)$

$a = \sqrt[3]{64x^3y^3} = 4xy$

$b = \sqrt[3]{125z^3} = 5z$

$(a-b)(a^2+ab+b^2)$

$= (4xy-5z)\left((4xy)^2 + (4xy)(5z) + (5z)^2\right)$

$= \boxed{(4xy-5z)(16x^2y^2 + 20xyz + 25z^2)}$

(5) $\frac{x^6}{27} + 8y^3$

- 2 terms
- added
- both are perfect cubes

Sum of Two Cubes

$a^3 + b^3 = (a+b)(a^2-ab+b^2)$

$a = \sqrt[3]{\frac{x^6}{27}} = \frac{x^2}{3}$

$b = \sqrt[3]{8y^3} = 2y$

$(a+b)(a^2-ab+b^2)$

$= \left(\frac{x^2}{3} + 2y\right)\left(\left(\frac{x^2}{3}\right)^2 - \left(\frac{x^2}{3}\right)(2y) + (2y)^2\right)$

$= \boxed{\left(\frac{x^2}{3} + 2y\right)\left(\frac{x^4}{9} - \frac{2x^2y}{3} + 4y^2\right)}$

(6) $36x^2 + 4$

$= \boxed{\text{PRIME}}$

- 2 terms
- added
- both are perfect squares

Sum of two squares is PRIME